Rotational Dynamics

⊛ Moment of inertia of rotating body :-

Let us consider a rigid body which consists of particles of masses $m_1, m_2, m_3 ---- m_n$ with distance $r_1, r_2, r_3 ---- r_n$ from axis of rotation AB. The moment of inertia of the body about axis AB is defined by,

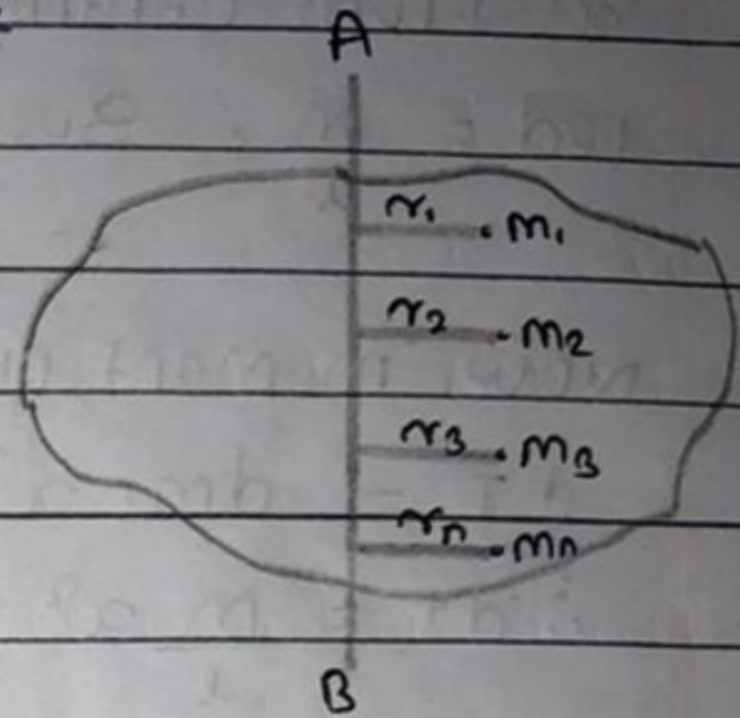

fig: A rigid body rotating

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + --- + m_nr_n^2$$

$$I = \Sigma mr^2$$

Thus, moment of inertia of a rigid body is defined as the sum of product of mass and square of distance from axis of rotation of indivisual particles.

⊛ Moment of inertia of uniform rod

① About the axis passing through centre and perpendicular to its length :-

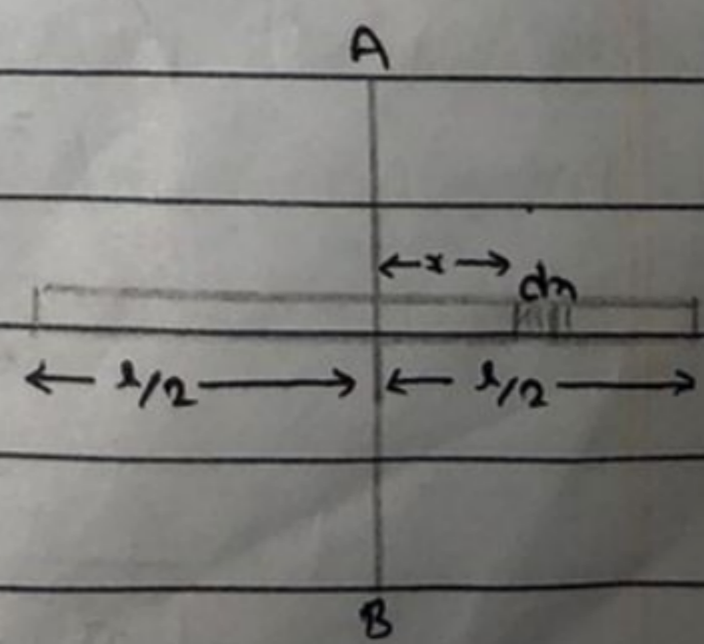

fig: moment of inertia of uniform rod.

Let us consider a uniform rod of length 'l' and mass 'm'. Let AB be the axis passing through centre and perpendicular to its length about which moment of inertia is to be determined.

Consider a small segment of length dx at distance 'x' from centre. Now mass per unit length of the rod $= \frac{m}{l}$. So, mass of the small segment $(dx) = \frac{m}{l} \cdot dx$

Now, moment of inertia of small segment about axis AB,

$$dI = dm \cdot x^2$$

$$\therefore dI = \frac{m}{l} x^2 \, dx \quad \text{--- (1)}$$

Thus, moment of inertia of whole rod about axis AB is obtained by integrating eqn (1) as,

$$I = \int_{-l/2}^{l/2} dI$$

$$= \int_{-l/2}^{l/2} \frac{m}{l} x^2 \, dx$$

$$= \int_{-l/2}^{l/2} \frac{m}{l} x^2 \, dx$$

$$= \frac{m}{l} \left[\frac{x^3}{3} \right]_{-l/2}^{l/2}$$

$$= \frac{m}{l} \left[\frac{(\frac{l}{2})^3}{3} - \frac{(-\frac{l}{2})^3}{3} \right]$$

$$= \frac{m}{l} \left[\frac{l^3}{24} + \frac{l^3}{24} \right]$$

$$\boxed{I = \frac{ml^2}{12}}$$

This is required expression for M.I of a uniform rod when the axis passing through centre and perpendicular to its length.