Periodic motion

Simple harmonic motion:

The motion of the body is said to be simple harmonic motion if its acceleration is directed towards the mean position and is directly proportional to the displacement from that position.

i.e In Simple Harmonic motion,

acceleration $\propto$ displacement

acceleration = $-k \times$ displacement

where k is constant and the negative sign shows that acceleration is directed in opposition to the motion of object.

Characteristics of Simple Harmonic motion (Relation between the acceleration and displacement of the particle executing S.H.M)

Let us consider a body having mass 'm' is moving in a circular path of radius 'r' with constant speed 'v'. Suppose body is initially at point A and after time 't' it reaches to point B describing angular displacement 'θ'. Draw perpendicular BN on verticle diameter YY'. Also, let when the particle at point B, the displacement by 'y' such that ON = y

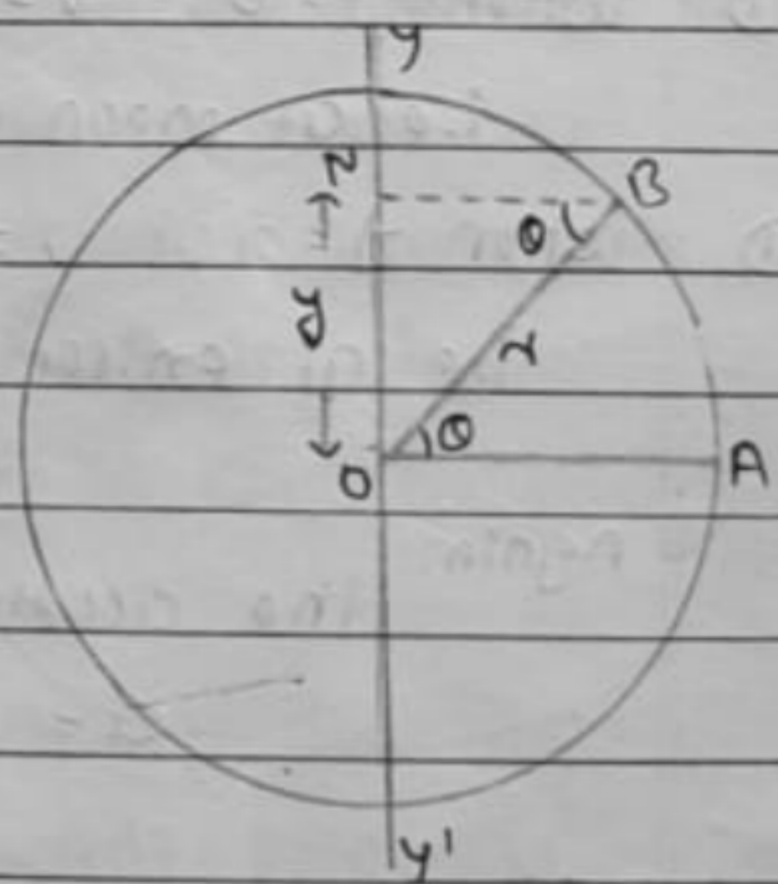

fig: body rotating in circular path

From figure,

$$\sin\theta = \frac{y}{r}$$

$$\text{or, } y = r\sin\theta \text{ — (i)}$$

If 'ω' be the angular velocity then $\theta = \omega t$

$$\therefore y = r\sin\omega t \text{ — (ii)}$$

This is the displacement equation for a particle of S.H.M

Now, velocity of S.H.M be.

$$v = \frac{dy}{dt}$$

$$= \frac{d(r\sin\omega t)}{dt}$$

$$\therefore v = r\omega\cos\omega t \text{ --- (iii)}$$

eq^n (iii) can also written as.

$$V = r\omega\sqrt{1-\sin^2\omega t}$$

$$v = \omega\sqrt{r^2 - r^2\sin^2\omega t}$$

$$v = \omega\sqrt{r^2 - y^2} \text{ --- (iv)}$$

This equation shows that velocity is not uniform

(i) when $y = 0$; $v = \omega r$

i.e at mean position the velocity is maximum.

(ii) when $y = r$; $v = 0$

i.e at extreme position, the velocity in minimum.

Again, the acceleration of S.H.M be,

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt} r\omega\cos\omega t$$

$$= r\omega^2 \sin\omega t$$

$$\boxed{a = -\omega^2 y} \text{ --- (v)}$$

This is the equation for the acceleration in S.H.M.

(i) when $y = 0$; $a = 0$

i.e at mean position the acceleration is zero.

(ii) when $y = r$; $a = -\omega^2 r$

i.e at extrem position the acceleration is maximum.