Simple Pendulum :-

A simple pendulum is a heavy point mass suspended by an inextensible, weightless and flexible string from a rigid support and which is face to oscillate in a verticle plane.

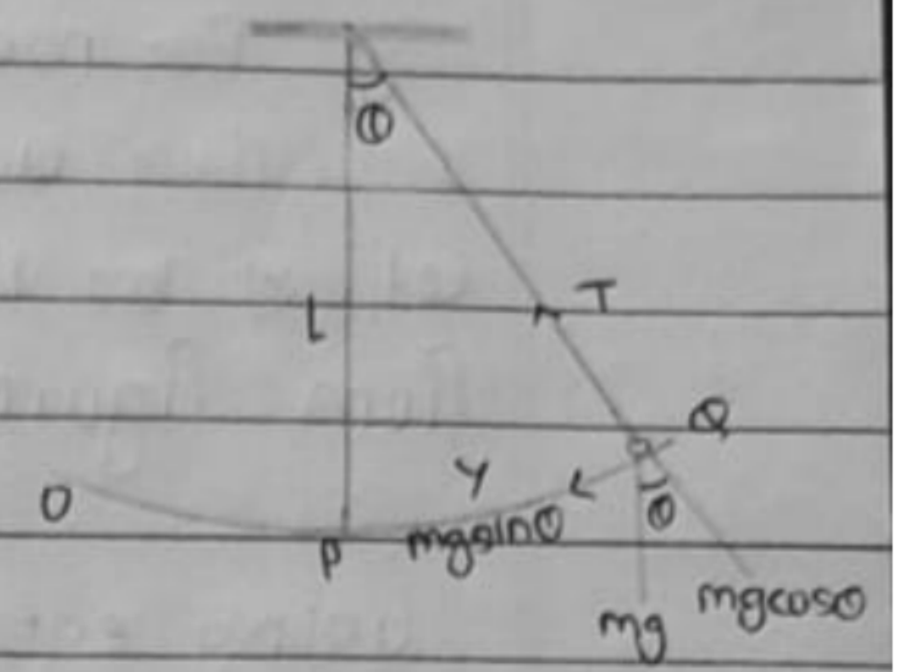

fig: simple pendulum

Let m be the mass of the bob of a simple pendulum and 'l' be it's effective length (distance between point of suspension to the c.g of the body) when the bob is not oscillating, the position of bob is called mean position which lies at point 'P' below the point of suspension.

When the bob is displaced from its mean position, it oscillates along the path OPQ. Also let at any instant it will be at point Q with angular displacement θ. The force on the bob at point Q are.

(i) weight mg of the bob acting vertically downward
(ii) Tension in the string acting towards point of suspension

The weight mg of the bob can be revolved into two components one is $mg\cos\theta$ opposite to the tension and other is $mg\sin\theta$ towards mean position. So $mg\cos\theta$ balance tension and $mg\sin\theta$ provides restoring force.

So, restoring force, $F = -mg\sin\theta$

for small θ

$$\sin\theta \simeq \theta$$

$$\therefore F = mg\theta \text{ --- } (1)$$

If a be the acceleration of the bob then,

$$F = ma = -mg\theta$$

$$a = -g\theta \quad \text{--- (ii)}$$

Let, y be the displacement from mean position then from figure, $\theta = \frac{y}{l}$ --- (iii)

using eq^n (iii) in (ii), we get

$$a = -g\frac{y}{l}$$

$$a = -\left(\frac{g}{l}\right)y \quad \text{--- (iv)}$$

Since $\frac{g}{l}$ is a constant for a given pendulum at a given place,

$$a \propto y$$

This shows that motion of simple pendulum is S.H.M

Also, we have acceleration in S.H.M be,

$$a = -\omega^2 y \quad \text{--- (v)}$$

Comparing eq^n (iv) and (v)

$$-\omega^2 = -\frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \quad \text{--- (vi)}$$

This is the required relation for time period of a simple pendulum and this relation shows that time period of a simple pendulum is independent to mass of the bob but depends upon the effective length and acceleration due to gravity at that place.

Q) what happens the time period of simple pendulum if it is taken to the top of mountain?

Soln Since, time period for simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, on the top of mountain the value of 'g' decreases and time period $T \propto \frac{1}{\sqrt{g}}$. Therefore the time period of simple pendulum increases when it is taken to the top of the mountain.

Oscillation of a loaded spring

ⓐ Vibration of a Particle in Horizontal Spring :-

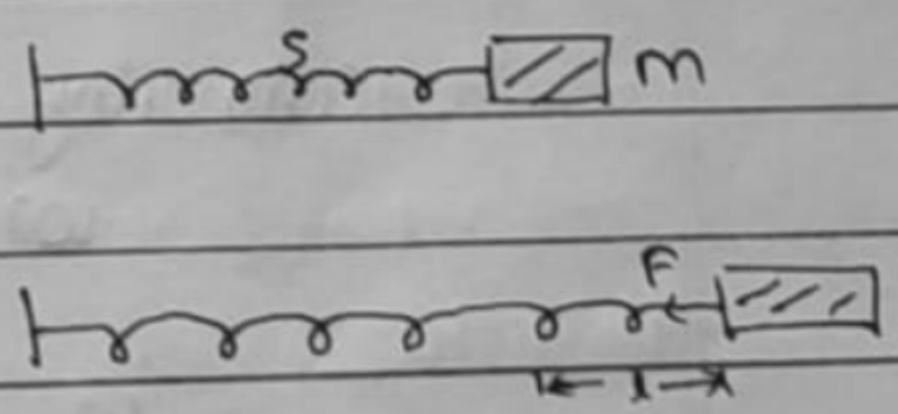

fig: Horizontal spring

Let us consider a spring of negligible mass whose one end is attached to wall other end is attached to an object of mass 'm'. The spring and the object lies on horizontal table as shown in figure. When the mass is pulled, the spring extends and let 'l' be the elognation produced. Then by HOOKS law restoring force set up in the spring be,

$$F \propto l$$

$$F = -kl \quad \text{—} ①$$

where k is constant called force constant of spring force per unit extension.

Negative sign shows that restoring force acts opposite to the displacement of mass.